BMED436 Medical Robot
2018250064 Sibeen Kim

Dynamic Analysis

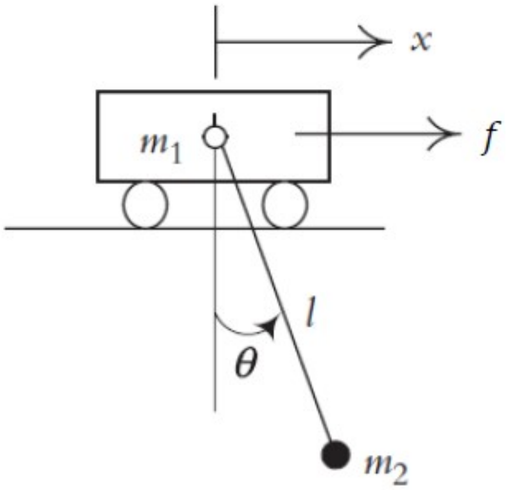

Figure 1: Cart-Pendulum System.

Figure 1 shows a cart-pendulum system. This setup consists of a cart that can move along a horizontal axis and a pendulum attached to it, which swings freely.

1 System Modeling

1.1 Equations of Motion

$$
\begin{aligned}
f &= (m_1 + m_2)\ddot{x} + m_2 l \cos\theta\ddot{\theta} - m_2 l \sin\theta\dot{\theta}^2 \\
&\approx (m_1 + m_2)\ddot{x} + m_2 l\ddot{\theta} \quad \text{(for small } \theta, \dot{\theta}\text{)}
\end{aligned}
$$

$$
\begin{aligned}
0 &= m_2 l \cos\theta\ddot{x} + m_2 l^2\ddot{\theta} + m_2 g l \sin\theta \\
&\approx m_2 l\ddot{x} + m_2 l^2\ddot{\theta} + m_2 g l\theta \quad \text{(for small } \theta\text{)}
\end{aligned}
$$

$$
f = (m_1 + m_2)\ddot{x} + m_2 l\ddot{\theta} \tag{1}
$$

$$
\ddot{x} = -l\ddot{\theta} - g\theta \tag{2}
$$

1.2 Transfer Function

From (1), (2):

$$
f = -m_1 l\ddot{\theta} - (m_1 + m_2)g\theta
$$

$$
\frac{\Theta(s)}{F(s)} = \frac{-1}{m_1 l s^2 + (m_1 + m_2)g} \tag{3}
$$

From (2):

$$
\frac{X(s)}{\Theta(s)} = -(ls^2 + g) \tag{4}
$$

2 Time Response

2.1 Simulink Implementation

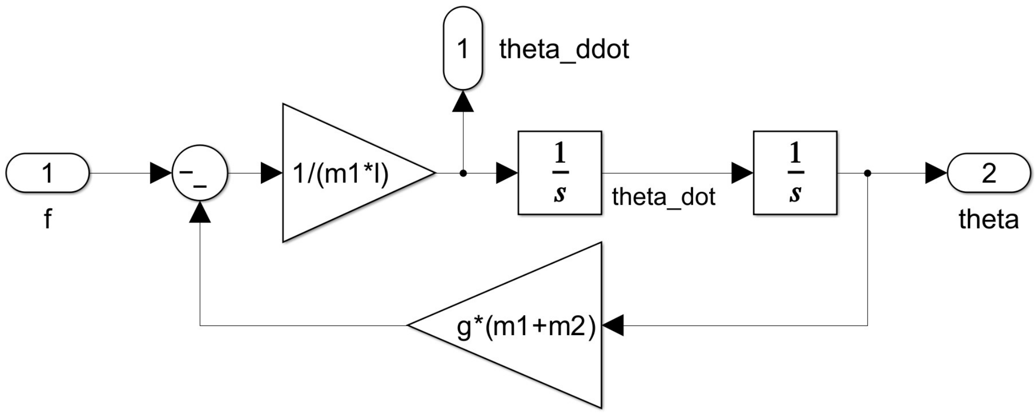

Figure 2: Plant Θ.

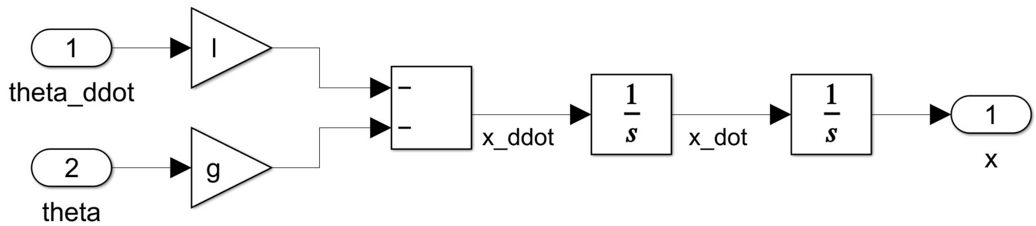

Figure 3: Plant X.

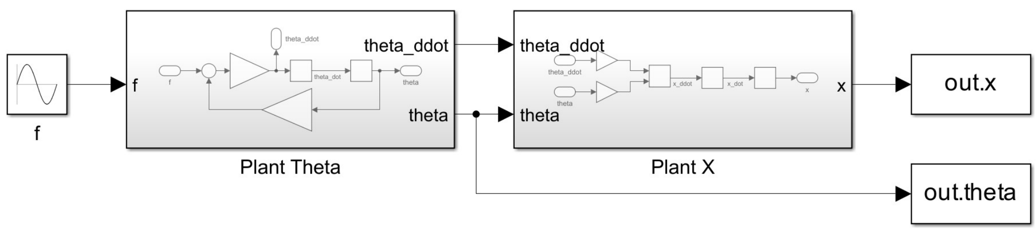

Figure 4: Overall system.

2.2 $\mathbf{q}(0) = \begin{bmatrix} 0 \\ -\frac{\pi}{3} \end{bmatrix}, \; \dot{\mathbf{q}}(0) = \mathbf{0}, \; f = 0$

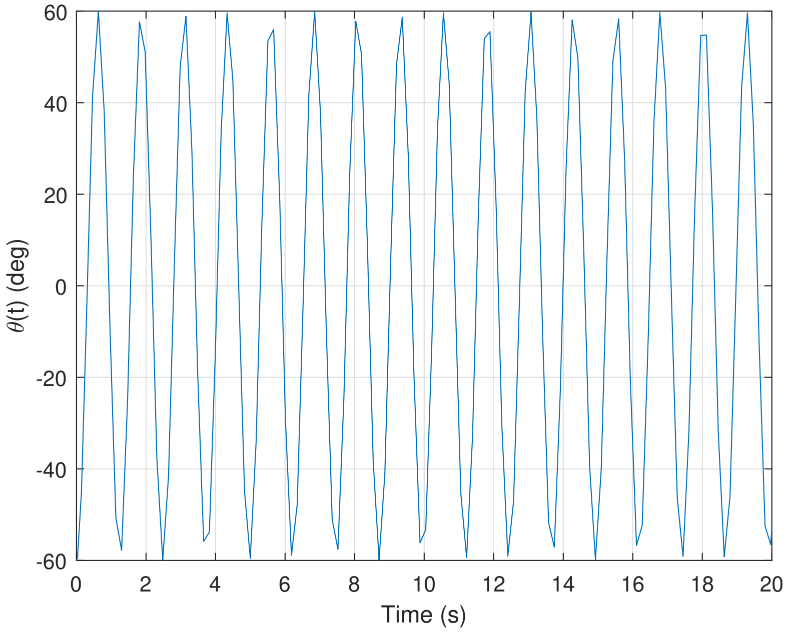

Figure 5: Time response: $\theta(t)$.

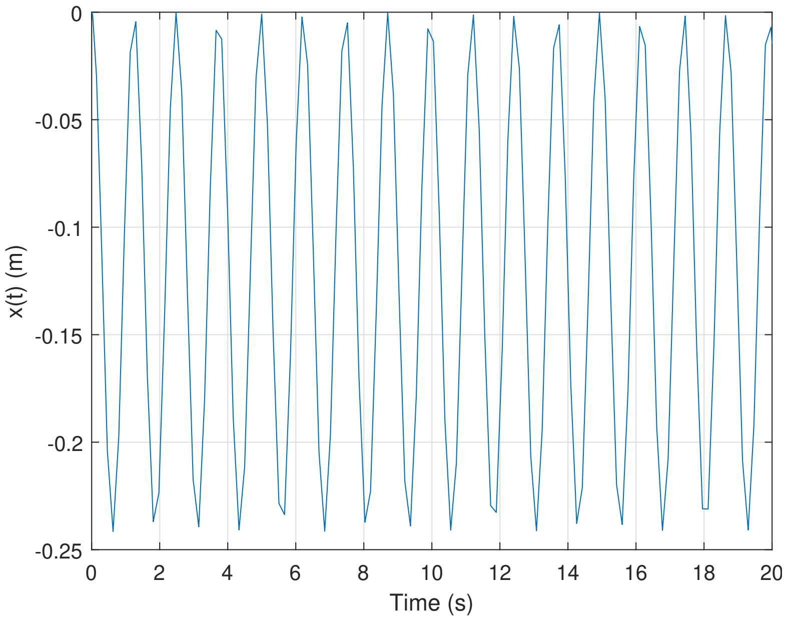

Figure 6: Time response: $x(t)$.

2.3 $\mathbf{q}(0) = \mathbf{0}$, $\dot{\mathbf{q}}(0) = \mathbf{0}$, $f = 2\sin \pi t$

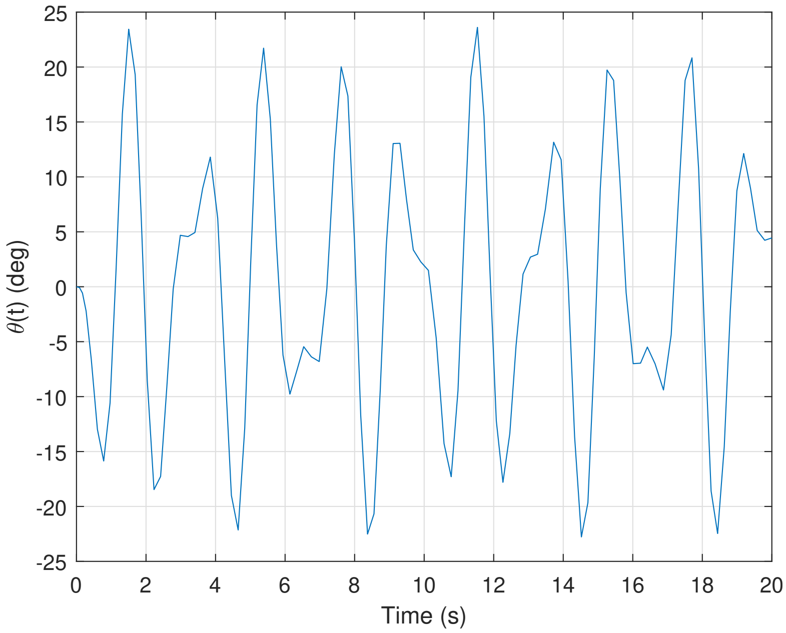

Figure 7: Time response: $\theta(t)$.

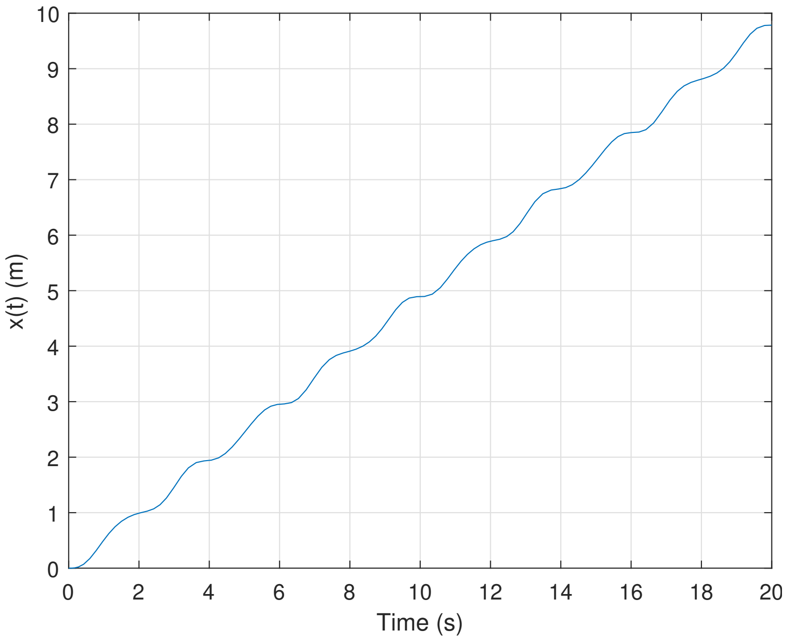

Figure 8: Time response: $x(t)$.